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V. V. Angeleiko and A. B. Kitsenko

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ABSTRACT

27725

The authors investigated the low-frequency electron-ion oscillations of a weakly ionized plasma. It is assumed that the equilibrium distribution of electrons satisfies by velocities the kinetic equation with the collision integral in Davydov's form (ref. 2). It is shown that the electron drift with respect to ions, in the presence of a strong external electric field, may lead to the instability of distribution obtained by Davydov. This instability is associated with the excitation of electron-ion oscillations.

Author

1. The equilibrium distribution of electrons by velocity in the weakly ionized plasma found in external electric and magnetic fields was studied by Druyvesteyn (ref. 1), Davydov (ref. 2), etc. It is a known fact that the electron drift in relation to the ions in an external electric field is conducive to the instability of the initial distribution. This brings up the question of the stability of the steady state solutions found in references 1 and 2. In

[1]Numbers given in the margin indicate the pagination in the original foreign text.

the absence of an external magnetic field, as shown by Akhiezer and Sitenko (ref. 3), the accretion of high-frequency disturbances with a Langmuir electron frequency is impossible. In the presence of both external electric and magnetic fields, the weakly ionized plasma is also stable in relation to high-frequency disturbances (ref. 4).

The waves whose phase velocity V_Φ is considerably higher than the electron thermal velocity v_e are discussed in references 3 and 4. This study deals with the propagation in a weakly ionized plasma of ion-sound waves with a phase velocity considerably lower than the thermal velocity of electrons but considerably higher than the thermal velocity of ions $v_i (v_i \ll V_\Phi \ll v_e)$.

As is known, the propagation of underdamped waves in this case is possible if the electron temperature is considerably higher than the ion temperature. The excitation of ion-sound waves in such an nonisothermal plasma by the electron drift in relation to the ions was examined in Gordeyev's study (ref. 5). It was assumed in the latter study that the equilibrium distribution of electrons by velocity was a Maxwellian distribution in a system of coordinates associated with electrons.

The equilibrium distribution function in this study was selected with a view to satisfying the kinetic equation with the collision integral in Davydov's form (ref. 2). As pointed out further on, the electron drift in relation to the ions in the external electric field may lead to the instability of the distribution obtained by Davydov.

2. The electron distribution function $F(\vec{r}, \vec{v}, t)$ satisfies the following

$$\frac{\partial F}{\partial t} + \vec{v} \operatorname{grad} F + \frac{e}{m} (\vec{E}_0 + \vec{E}) \frac{\partial F}{\partial \vec{v}} + J(F) = 0, \qquad (1)$$

where $\vec{E}_0$ is the external electric field, $\vec{E}$ the electric field produced by oscillations, and J(F) the collision integral in Davydov's form (ref. 2). In a strong external electric field where the effective electron temperature is considerably higher than the ion temperature, the equilibrium electron distribution function by velocity, in the absence of oscillations, looks like the following: /17

$$F_0 = f_0(v^2) + \vec{E}_0 \vec{v} f_1(v^2),$$
$$f_0(v^2) = C \exp\left\{-3 \frac{m^3}{e^2 E_0^2 M} \int^{v} \frac{v^3}{l^2(v)} dv\right\}, \tag{2}$$
$$f_1(v^2) = -\frac{el}{mv^2} \frac{df_0}{dv},$$

where l is the length of the electron free path, m and M are the electron and ion masses, and C the normalization constant. Formulas (2) can be used in the case of a strong external electric field if the inequality $\xi = \frac{M}{6m}\left(\frac{eE_0 l}{T}\right)^2 \gg 1$, in which T denotes the ion temperature, is satisfied. In the case of numerous types of gases it may be roughly assumed that the length of the free path does not depend on the velocity. In this case we have (ref. 1)

$$f_0 = C \exp\left(-\frac{3m^3 v^4}{4Me^2 E_0^2 l^2}\right). \tag{3}$$

Expanding the small disturbances of the distribution function $f = F - F_0$ and electric field $\vec{E}$ on plane waves $\exp i(\vec{k}\vec{r} - \omega t)$ we will get the following equation

$$-i\omega f + i\vec{k}\vec{v} f + \frac{e}{m} \vec{E} \frac{\partial F_0}{\partial \vec{v}} + \frac{e}{m} \vec{E}_0 \frac{\partial f}{\partial \vec{v}} = J(f), \tag{4}$$

where $\vec{k}$ and ω are the wave vector and wave frequency.

If the phase velocity of wave $V_{\Phi} = \frac{\omega}{k}$ is considerably greater than the thermal velocity of the ions, the movement of the ions can be described by the use of hydrodynamic equations. The ion-molecule collision may be taken into account if a frictional force proportional to the ion-molecule collision frequency v_i is included in the movement equation. The directed velocity of the ions is in the order of $\frac{eE_0}{Mv_i}$ and cannot be much greater than the thermal speed of the ions.

When studying the waves with $V_{\Phi} \gg v_i$, both the thermal movement of the ions and the ion drift in the external electric field may be disregarded. To find the equation for the longitudinal electron ion oscillations, we can also use the following equation

$$\operatorname{div} E = 4\pi\rho \tag{5}$$

where ρ is the total electron and ionic charge.

We will solve equation (4) by the successive approximation method, discarding in zero approximation the terms $\frac{e}{m}\vec{E}_0\frac{\partial f}{\partial \vec{v}}$ and $J(f)$. The result will be the following dispersion equation

$$1-\frac{\Omega_i^2}{\omega(\omega+i\nu_i)}+\frac{\Omega_e^2}{\omega n_0 k^2}\int\frac{(\vec{k}\vec{v})\left(\vec{k}\frac{\partial F_0}{\partial \vec{v}}\right)}{\omega-\vec{k}\vec{v}}\,d\vec{v}-i\frac{\Omega_e^2}{\omega n_0 k^2}\frac{e}{m}\int\frac{(\vec{k}\vec{v})}{\omega-\vec{k}\vec{v}}\times$$
$$\times\vec{E}_0\frac{\partial}{\partial \vec{v}}\left(\frac{\vec{k}\frac{\partial f_0}{\partial \vec{v}}}{\omega-\vec{k}\vec{v}}\right)d\vec{v}+i\frac{\Omega_e^2}{\omega n_0 k^2}\int\frac{(\vec{k}\vec{v})}{\omega-\vec{k}\vec{v}}J\left(\frac{\vec{k}\frac{\partial f_0}{\partial \vec{v}}}{\omega-\vec{k}\vec{v}}\right)d\vec{v}=0, \tag{6}$$

where n_0 is the equilibrium electron (ion) density; Ω_e and Ω_i the Langmuir electronic and ionic frequency. If the length of the electron free path does /18

not depend on the velocity, the expression (3) can be used for f_0. Assuming that $V_\Phi \ll v_e$, we can use (6) to obtain

$$1-\frac{\Omega_i^2}{\omega(\omega+i\nu_i)}+\frac{\Omega_i^2}{k^2V_s^2}\left[1+i\pi\frac{2^{1/4}}{\Gamma\left(\frac{1}{4}\right)}\frac{\omega}{kv_e}-i\pi\frac{\Gamma\left(\frac{3}{4}\right)}{\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)}\frac{eE_0l}{mv_e^2}\cos\theta+\right. \tag{7}$$

$$\left.+a\frac{\nu_i}{\omega}\sqrt{\frac{m}{M}}+ib\frac{\nu_e}{\omega}\frac{m}{M}\right]=0,$$

where v_e is the electron-molecule collision frequency; $v_e=\xi^{\frac{1}{4}}\sqrt{\frac{T}{m}}$; $V_s^2=$ $=v_e^2\frac{m}{M}\frac{2^{1/4}\Gamma(^3/_4)}{\Gamma(^1/_4)}$; Γ the gamma-function; Θ the angle between $\vec{E}_0$ and $\vec{k}$; α and b the actual unit constants.

It follows from (7) that in order to use the successive approximation method to solve equation (4), we must see to it that the following inequality is complied with $\frac{\nu_e}{\omega}\sqrt{\frac{m}{M}}\ll 1$. The solution of equation (7) has the form

$$\omega=\omega_0+i\gamma,\qquad \omega_0=\frac{kV_s}{\sqrt{1+k^2a_e^2}}. \tag{8}$$

$$\gamma=\gamma_L-\gamma_s,$$

$$\frac{\gamma_L}{\omega_0}=-\pi\frac{2^{-3/4}}{\Gamma\left(\frac{1}{4}\right)}\frac{\omega_0}{kv_e}+\pi\frac{2^{-3/2}\Gamma\left(\frac{3}{4}\right)\cos\theta}{\Gamma\left(\frac{1}{4}\right)}\sqrt{\frac{6m}{M}}. \tag{9}$$

Here α_e $\frac{V_s}{\Omega_i}$ represents the Debye radius of the electrons, $\gamma_s \approx \nu_i + \nu_e\frac{m}{M}$ the extinction factor determined by the electron and ion collisions with molecules, and γ_L the Landau intensification (extinction). We should point out that, unlike the high-frequency oscillations, the extinction of the ion-sound oscillations produced by the electron-molecule collisions is defined not by collision frequency ν_e, but by an $\frac{m}{M}$ times smaller magnitude (ref. 6). If follows from (8)

and inequality $\frac{\nu_e}{\omega}\sqrt{\frac{m}{M}} \ll 1$ that the investigated wavelength should satisfy the condition $\frac{\lambda}{2\pi} \ll 1$. With adequately large wavelength $\left(\frac{\lambda}{2\pi} \gg \alpha_e\right)$ we can find the following from (8)

$$\omega_0 = kV_s,$$

$$\frac{\gamma_L}{\omega_0} = \frac{\Gamma^2\left(\frac{3}{4}\right)}{\sqrt{\pi}2^{3/4}}\sqrt{\frac{m}{M}}\left(\frac{\sqrt{6\pi}}{2^{5/4}}\cos\theta - 1\right). \tag{10}$$

The solution of equation (6), assuming an arbitrary relationship between the length of the electron free path and its velocity, takes the form (8) in which the magnitudes γ_L and V_s are defined by the following formulas /19

$$\frac{\gamma_L}{\omega_0} = -\frac{\pi^2\omega_0^2 M}{n_0 k^2 m}\left(\frac{\omega_0}{k} f_0'(0) + \frac{eE_0}{m}\cos\theta \int_0^\infty \frac{l(u)}{u}\frac{\partial f_0}{\partial u}du\right),$$

$$V_s^2 = \frac{n_0}{2\pi \int_{-\infty}^{\infty} f_0(u^2)\,du}. \tag{11}$$

and function f is found from (2).

The excitation of oscillations occurs in the case of the following inequality: $\gamma_L > \gamma_s$. If the length of the free path does not depend on the velocity, the excitation of "sound" waves $\left(\frac{\lambda}{2\pi} \gg \alpha_e\right)$ is observable when the angle between the direction of the wave propagation and the external electric field satisfies condition $\cos\Theta > \frac{2^{5/4}}{\sqrt{6\pi}}$. The following inequality is valid for the frequency of the investigated oscillations $\nu_i,\ \nu_e\sqrt{\frac{m}{M}} \ll \omega < \Omega_i$. This inequality is fulfilled only in case the Debye radii of the electrons α_e and ions $\alpha_i = \frac{v_i}{\Omega_i}$ are small in comparison with the length of the free path of the electrons and ions $l_i = \frac{v_i}{\nu_i}$ $(a_e \ll l_e,\ a_i \ll l_i)$, respectively.

3. Let us examine the excitation of electron-ion oscillations in a weakly ionized plasma contained in strong external electric and magnetic fields. We will assume that the external magnetic field $\vec{H}_0$, directed along a z axis, is permanent and homogeneous. This assumption is valid if the azimuthal magnetic field H_φ, which is produced by a current passing through the plasma, is small $\frac{H_\varphi}{H_0} \ll 1$. If the wavelengths are small in comparison with the dimensions of the system, this condition will lead to the following inequality

$$\frac{u_0}{V_A}\frac{\omega_{Hi}}{kV_A} \ll 1, \tag{12}$$

where ω_{Hi} is the cyclotron frequency of the ions, u_0 the velocity of the electron drift and V_A the Alfvenic velocity.

We will note that $u_0 \sim V_s \sim v_e \sqrt{\frac{m}{M}}$. It follows from (12) that the excitation of magnetohydrodynamic waves ($\omega \ll \omega_{Hi}$) with a phase velocity considerably greater than or about equal to an Alfvenic velocity is impossible. Actually, according to (12), $u_0 \ll V_A$, and the excitation of waves with $V_\Phi \geqslant V_A$ requires high velocities and a drift ($u_0 \geqslant V_A$). In the low-frequency ($\omega \ll \omega_{Hi}$) region, however, it is possible to excite slow "sound" oscillations with a phase /20
velocity of $V_\Phi \sim u_0 \sim V_s \ll V_A$. With $\nu_i, \nu_e \sqrt{\frac{m}{M}} \ll \omega \ll \omega_{Hi}$, it is possible to use formulas (8)-(11) for the oscillation frequency and increment, in which case the only requirement is the substitution of E_{0z} for E_0, k_z for k and 1 for $\cos\theta$.

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REFERENCES

1. Druyvesteyn, M. J. Physica, 10, 61, 1930; 1, 1003, 1964.

2. Davydov, B. I. Journal of Experimental and Theoretical Physics (ZhETF) 6, 463, 1936; 7, 1069, 1937.

3. Akhiezer, A. I. and Sitenko, A. G. ZhETF, 30, 216, 1956.

4. Stepanov, K. N. and Tkalich, V. S. ZhETF, 28, 1789, 1958.

5. Gordeyev, G. F. ZhETF, 27, 19, 24, 1954.

6. Gertzenshteyn, M. Ye. ZhETF, 24, 652, 1953.

Physico-Technical Institute
Ukrainian Academy of Sciences,
Kharkov.

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